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Convective drying of a consolidated slab of wet porous material

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Abstract—A model is formulated to describe the drying process of a consolidated finite slab of wet porous material by forced convection of hot, dry air past the exposed surface of a slab. The model is very comprehensive and describes the evolution of temperature, pressure and moisture distributions in both the wet and dry regions. The distinct advantage of the model is that the coefficients are constructed from well-known experimental results. The model is illustrated by application to the drying of wet brick. The system of three non-linear coupled partial differential equations describing the physical process, is solved numerically using finite difference techniques.

1. INTRODUCTION

THE FIRST half of this century saw several theories to explain the various stages of moisture migration in porous media. These included the Diffusion Theory [1–4], the Capillary Flow Theory [5], and the Evaporation–Condensation Theory [6]. During the later part of the century, more comprehensive theories [7–11] of simultaneous heat and mass transfer processes have been advanced. The starting point for these theories is either the phenomenological approach (Luikov's School), dealing directly with averaged quantities and experimentally determined coefficients [12–14], or continuum point equations which are averaged (Whitaker's School) [15–22].

In this study the starting point is the integral form of the conservation of mass and energy. The quantities used throughout the analysis are the intrinsic phase averaged quantities defined at the appropriate level [23], and assumed to be sufficiently smooth to allow all mathematical manipulations. These integral forms are then reduced to partial differential equations in the usual manner. The quantities corresponding to different phases are separated by simply identifying one expression as the source/sink for the other. The equations so obtained can be related to both Luikov's and Whitaker's equations [23].

We believe that our formulation has a number of advantages. Conceptually it is simple and can be understood by any student of continuum mechanics. The paper is self-contained: we begin with straightforward conservation equations and finish with three precisely defined partial differential equations. There is no need for complicated background theory on averaging like Whitaker's theory nor any knowledge of irreversible thermodynamics like Luikov's theory. The 'kinetic' coefficients such as K_S, K_e, K_P and K_T are determined explicitly by the analysis and it is not necessary to obtain them from experimental results. The source term, $\dot{m}$, has been eliminated which is still present in ref. [20]. Our model includes not only the three drying dynamics variables—moisture content, temperature and pressure—but also water vapour pressure both above equilibrium moisture content (e.m.c.) when it is given by the Clapeyron equation and below e.m.c. when it is determined as one of the variables of the moving boundary problem. Knowledge of all these variables is required for proper quality control of the final dried product.

The real advance on the existing theories becomes apparent when the theory is applied to 'non-equilibrium' processes, i.e. processes when the temperature of the solid–water phase is different from the temperature of the binary mixture (air and steam). A family of 'practical' problems that fall into this category are industrial processes involving through (or cross) flow such as drying of iron ore pellets which we discussed in ref. [23]. The original purpose of the present paper was to calculate the drying kinetics of individual pellets and hence obtain an expression for the source term which was discussed in ref. [23]. It became clear that the method is equally applicable to a pellet, brick, concrete slab or whatever if the same averaging volume can be used. Another family of 'practical' problems that fall into this category are those that involve microwave drying which is being

NOMENCLATURE

C_p	specific heat [J kg^{-1} K^{-1}]
D	diffusivity [m^2 s^{-1}]
g	gravitational constant [m s^{-2}]
h	intrinsic averaged enthalpy [J kg^{-1}]
h_{vap}	latent heat of evaporation [J kg^{-1}]
J	Leverett function
k	thermal conductivity [W m^{-1} K^{-1}]
k_m	mass transfer coefficient [m s^{-1}]
K	intrinsic permeability [m^2]
$K_{\mathrm{g}}, K_{\mathrm{w}}$	relative permeabilities of gas and water
L	thickness of porous medium [m]
M	molar mass [kg mol^{-1}]
$\mathbf{n}$	unit outward normal
P	pressure [Pa]
$\tilde{P}$	non-dimensional pressure
Q	heat transfer coefficient [W m^{-2} K^{-1}]
R	universal gas constant [J mol^{-1} K^{-1}]
S	volume saturation
S'	surface saturation
T	temperature [K]
t	time [s]
$\mathbf{v}$	averaged velocity [m s^{-1}]
x	distance [m]
x'	non-dimensional distance.

Greek symbols

β	surface tension constant [N m^{-1} K^{-1}]
Θ	non-dimensional temperature
μ	dynamic viscosity [kg m^{-1} s^{-1}]
ρ	intrinsic averaged density [kg m^{-3}]
σ	surface tension [N m^{-1}]
σ_0	surface tension constant [N m^{-1}]
τ	non-dimensional time
ϕ	porosity [m^3 m^{-3}]
χ	surface fraction
χ	surface porosity, $\chi_{\mathrm{g}}+\chi_{\mathrm{w}}$ [m^2 m^{-2}].

Subscripts

a	air
c	capillary
cr	critical
g	gas
i	spatial index
ir	irreducible
s	solid
v	vapour
w	liquid
0	atmospheric
1	initial.

examined by Turner. We see a place for this paper in the literature not only for its importance in further development but also for establishing common ground with existing theories.

We believe that the most significant advance is still to come. Since our theory is basically mechanical rather than thermodynamical like Luikov's (see preface of ref. [12]), it can be extended to include stress distribution in a porous medium whereas in Luikov's formulation stresses are calculated by postulating an empirical stress–temperature relation [14] or stress–moisture relation [31]. This can be done by using the conservation of momentum equations and the structure of the porous medium instead of Darcy's law and the results connected with it (permeability, capillary pressure, etc.) which are used in this paper.

Apart from the basic equations, another essential ingredient is to recognize when the mathematical variables are physically valid. This has lead to so many subdivisions of the drying process: saturated, unsaturated, constant rate, falling rate, above and below equilibrium moisture content, above and below irreducible saturation and so on.

Most of these regimes can be expressed by inequalities involving saturation, S. For example if S_0 is the equilibrium moisture content, then when $S_0 < S < 1$ it is believed that liquid and vapour are in equilibrium, the relevant relation being the Clausius–Clapeyron equation or a modified version of it [8, 17]. When $0 < S < S_0$ this relation is no longer valid and when $S = 0$, the material is dry. Similarly if S_{ir} is the irreducible saturation, Darcy's law holds when $S_{\mathrm{ir}} < S < 1$ but there is no water flow when $0 < S < S_{\mathrm{ir}}$. Such problems are not easily incorporated into the usual analysis since they are basically moving boundary value problems [17].

In the present paper we assume that there is a critical value of S_{ir}, above which both Clapeyron's and Darcy's laws are valid. Further we assume that this value is small enough ($S_{\mathrm{ir}} = 0.09$ for brick) so that for most computing work this number is close to zero and when $S = S_{\mathrm{ir}}$ we take the medium to be dry. To put it another way, it is assumed that the region $0 < S < S_{\mathrm{ir}}$ is fairly narrow and can be taken to constitute an averaged surface (line) jump [17]. Thus, in the present model there are two regions: the wet region and the dry region, separated by $S = S_{\mathrm{ir}}$; we start with the wet region and finish with the dry region.

The case of a wet and dry region under various assumptions has been solved by the moving boundary value problem with certain jump conditions at the evaporation front [17, 23–25]. For the finite difference approximations, these jump conditions need not be stated explicitly; all that is required is certain continuity conditions which will be discussed later.

Convective drying plays an important role in many industrial processes. Although the design of driers varies from one industry to the next the central prob-

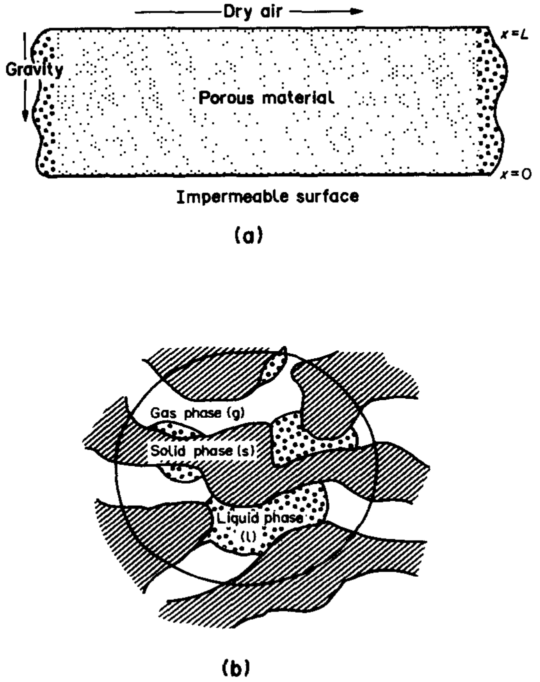

FIG. 1. (a) Configuration of physical problem, (b) Macroscopic averaging volume element.

lem is to be able to predict the temperature, moisture content and pressure profiles within the material being dried. Our contribution consists in the theoretical study of an idealized one-dimensional heat and mass transfer during convective drying of a consolidated slab of a wet porous material. The drying process under investigation is illustrated in Fig. 1(a) where the shaded region $0 \leqslant x \leqslant L$ represents the porous slab being dried by forced convection of hot dry air ($x > L$). In this paper it is assumed that the results will not depend significantly on y and z whichever unit cross-sectional area in the y–z plane is taken. Within each macroscopic average volume of the porous slab, Fig. 1(b), there is a solid phase which has constant density, porosity and is inert and rigid; a liquid phase (water) and a gaseous phase containing both air and water vapour. The face $x = L$ of the slab is exposed to an air flux with fixed characteristics (temperature, velocity and humidity), the other is adiabatic and impervious. Initially the slab is isothermal and at hydrostatic equilibrium with $S > S_{\mathrm{ir}}$.

This statement of the problem is identical to the one found in refs. [17, 20]. We also use the controlling input constants found in ref. [20] in numerical results for easier comparison with that work.

Further assumptions are given below.

(i) Both the solid and gas phases are continuous as is the liquid phase in the wet region; water is not bound to the solid.

(ii) Binary mixture of air and steam behaves like an ideal gas.

(iii) There is a local equilibrium within each macroscopically small averaging volume.

(iv) Darcy's law holds for the gas and liquid phases.

(v) Gravity is important for liquid but not for the gas phase.

(vi) Conduction occurs within each phase; the thermal conductivity of each phase is constant.

(vii) The enthalpy in the three phases is a linear function of temperature.

(viii) For $S > S_{\mathrm{ir}}$, water vapour pressure is given by the Clausius–Clapeyron equation.

(ix) Capillary pressure $P_c(S, T)$ is given by Leverett's formula [26].

(x) Latent heat h_{vap} is constant. Specific heat of air and steam are equal, i.e. $C_{pa} = C_{pv} = C_{pg}$. These are not essential to the model, but they do simplify some of the coefficients.

(xi) Permeabilities of gas and water can be expressed in terms of relative permeabilities.

Other simplifications are made in the text.

After the mathematical formulation of the problem in the next section, the system is simplified to the one-dimensional case and three non-linear, coupled partial differential equations for the wet and dry regions with relevant boundary conditions in the following section. The equations are nondimensionalized in the subsequent section and solved numerically by the finite difference method discussed in Section 5.

Then there follows a discussion of the results together with graphical representations, comparing them with experimental and theoretical results obtained elsewhere [20]; one of the highlights being Fig. 8, where the position of the moving evaporation front is plotted. We have not exhibited overall drying kinetics since they are similar to those found in ref. [20]. We look briefly at some simplified models proposed by Whitaker [16, 17], which are readily obtained from our formulation.

2. MATHEMATICAL FORMULATION OF PROBLEM

In order to make the derivation concise, the conservation laws are considered in their integral form. In an averaging volume element $\mathrm{d}V$ there is a binary mixture of air and steam (gas), porous material (solid) and water.

The mass and enthalpy in element $\mathrm{d}V$ are given respectively by

$$\mathrm{d}M = \phi S_{\mathrm{gv}}\rho_{\mathrm{g}}\,\mathrm{d}V + \phi S_{\mathrm{ga}}\rho_{\mathrm{g}}\,\mathrm{d}V + \phi S_{\mathrm{w}}\rho_{\mathrm{w}}\,\mathrm{d}V + (1-\phi)\rho_{\mathrm{s}}\,\mathrm{d}V$$

$$\mathrm{d}h = \phi S_{\mathrm{gv}}\rho_{\mathrm{g}}h_{\mathrm{gv}}\,\mathrm{d}V + \phi S_{\mathrm{ga}}\rho_{\mathrm{g}}h_{\mathrm{ga}}\,\mathrm{d}V + \phi S_{\mathrm{w}}\rho_{\mathrm{w}}h_{\mathrm{w}}\,\mathrm{d}V + (1-\phi)\rho_{\mathrm{s}}h_{\mathrm{s}}\,\mathrm{d}V.$$

The mass flow rate and enthalpy flux of steam through an averaging surface element $\mathrm{d}\sigma$ are given respectively by

$$\mathbf{v}_{\mathrm{gv}}\cdot\mathbf{n}\rho_{\mathrm{g}}\chi_{\mathrm{gv}}\,\mathrm{d}\sigma \quad \text{and} \quad \mathbf{v}_{\mathrm{gv}}\cdot\mathbf{n}\rho_{\mathrm{g}}\chi_{\mathrm{gv}}h_{\mathrm{gv}}\,\mathrm{d}\sigma.$$

The corresponding values for air and water are

$$\mathbf{v}_{\mathrm{ga}}\cdot\mathbf{n}\rho_{\mathrm{g}}\chi_{\mathrm{ga}}\,\mathrm{d}\sigma,\quad \mathbf{v}_{\mathrm{ga}}\cdot\mathbf{n}\rho_{\mathrm{g}}\chi_{\mathrm{ga}}h_{\mathrm{ga}}\,\mathrm{d}\sigma$$

$$\mathbf{v}_{\mathrm{w}}\cdot\mathbf{n}\rho_{\mathrm{w}}\chi_{\mathrm{w}}\,\mathrm{d}\sigma,\quad \mathbf{v}_{\mathrm{w}}\cdot\mathbf{n}\rho_{\mathrm{w}}\chi_{\mathrm{w}}h_{\mathrm{w}}\,\mathrm{d}\sigma.$$

The heat flux through an averaging surface element $\mathrm{d}\sigma$ is given by

$$\begin{aligned}&k_{\mathrm{g}}\mathbf{n}\cdot\nabla T_{\mathrm{g}}\chi_{\mathrm{g}}\,\mathrm{d}\sigma+k_{\mathrm{w}}\mathbf{n}\cdot\nabla T_{\mathrm{w}}\chi_{\mathrm{w}}\,\mathrm{d}\sigma\\&\quad+k_{\mathrm{s}}\mathbf{n}\cdot\nabla T_{\mathrm{s}}(1-\chi)\,\mathrm{d}\sigma\\&\quad=(k_{\mathrm{g}}\chi_{\mathrm{g}}+k_{\mathrm{w}}\chi_{\mathrm{w}}+k_{\mathrm{s}}(1-\chi))\mathbf{n}\cdot\nabla T\,\mathrm{d}\sigma.\end{aligned}$$

For a fixed volume V enclosed by a surface A, the conservation of mass and energy are

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\iiint_V(\phi S_{\mathrm{g}}\rho_{\mathrm{g}}+\phi S_{\mathrm{w}}\rho_{\mathrm{w}})\,\mathrm{d}V\\&\quad=-\iint_A\mathbf{v}_{\mathrm{g}}\cdot\mathbf{n}\rho_{\mathrm{g}}\chi_{\mathrm{g}}\,\mathrm{d}\sigma-\iint_A\mathbf{v}_{\mathrm{w}}\cdot\mathbf{n}\rho_{\mathrm{w}}\chi_{\mathrm{w}}\,\mathrm{d}\sigma\end{aligned}\tag{1}$$

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}t}\iiint_V(\phi S_{\mathrm{gv}}\rho_{\mathrm{g}}h_{\mathrm{gv}}+\phi S_{\mathrm{ga}}\rho_{\mathrm{g}}h_{\mathrm{ga}}\\&\quad+\phi S_{\mathrm{w}}\rho_{\mathrm{w}}h_{\mathrm{w}}+(1-\phi)\rho_{\mathrm{s}}h_{\mathrm{s}})\,\mathrm{d}V\\&=-\iint_A\mathbf{v}_{\mathrm{gv}}\cdot\mathbf{n}\rho_{\mathrm{g}}h_{\mathrm{gv}}\chi_{\mathrm{gv}}\,\mathrm{d}\sigma\\&\quad-\iint_A\mathbf{v}_{\mathrm{ga}}\cdot\mathbf{n}\rho_{\mathrm{g}}h_{\mathrm{ga}}\chi_{\mathrm{ga}}\,\mathrm{d}\sigma\\&\quad-\iint_A\mathbf{v}_{\mathrm{w}}\cdot\mathbf{n}\rho_{\mathrm{w}}h_{\mathrm{w}}\chi_{\mathrm{w}}\,\mathrm{d}\sigma\\&\quad+\iint_A(k_{\mathrm{g}}\chi_{\mathrm{g}}+k_{\mathrm{w}}\chi_{\mathrm{w}}+k_{\mathrm{s}}(1-\chi))\mathbf{n}\cdot\nabla T\,\mathrm{d}\sigma\end{aligned}\tag{2}$$

respectively, where we have set

$$S_{\mathrm{gv}}+S_{\mathrm{ga}}=S_{\mathrm{g}},\quad \chi_{\mathrm{gv}}\rho_{\mathrm{g}}\mathbf{v}_{\mathrm{gv}}+\chi_{\mathrm{ga}}\rho_{\mathrm{g}}\mathbf{v}_{\mathrm{ga}}=\chi_{\mathrm{g}}\rho_{\mathrm{g}}\mathbf{v}_{\mathrm{g}}.\tag{3}$$

By applying the divergence theorem and noting that V is arbitrary, the conservation laws are obtained in their differential form as

$$\begin{aligned}\frac{\partial}{\partial t}(\phi S_{\mathrm{g}}\rho_{\mathrm{g}})+\nabla\cdot(\chi_{\mathrm{g}}\rho_{\mathrm{g}}\mathbf{v}_{\mathrm{g}})&=-\frac{\partial}{\partial t}(\phi S_{\mathrm{w}}\rho_{\mathrm{w}})\\&\quad-\nabla\cdot(\chi_{\mathrm{w}}\rho_{\mathrm{w}}\mathbf{v}_{\mathrm{w}})\end{aligned}\tag{4}$$

$$\begin{aligned}&\frac{\partial}{\partial t}(\phi S_{\mathrm{gv}}\rho_{\mathrm{g}}h_{\mathrm{gv}}+\phi S_{\mathrm{ga}}\rho_{\mathrm{g}}h_{\mathrm{ga}}+\phi S_{\mathrm{w}}\rho_{\mathrm{w}}h_{\mathrm{w}}+(1-\phi)\rho_{\mathrm{s}}h_{\mathrm{s}})\\&\quad+\nabla\cdot(\chi_{\mathrm{gv}}\rho_{\mathrm{g}}\mathbf{v}_{\mathrm{gv}}h_{\mathrm{gv}}+\chi_{\mathrm{ga}}\rho_{\mathrm{g}}\mathbf{v}_{\mathrm{ga}}h_{\mathrm{ga}}+\chi_{\mathrm{w}}\rho_{\mathrm{w}}\mathbf{v}_{\mathrm{w}}h_{\mathrm{w}})\\&\quad=\nabla\cdot((k_{\mathrm{g}}\chi_{\mathrm{g}}+k_{\mathrm{w}}\chi_{\mathrm{w}}+k_{\mathrm{s}}(1-\chi))\nabla T).\end{aligned}\tag{5}$$

The liquid and gas phases in the mass equation are separated by identifying one expression as the source/sink for the other. Equation (4) gives

$$\frac{\partial}{\partial t}(\phi S_{\mathrm{g}}\rho_{\mathrm{g}})+\nabla\cdot(\chi_{\mathrm{g}}\rho_{\mathrm{g}}\mathbf{v}_{\mathrm{g}})=\dot{m}\text{ (Source)}\tag{6}$$

$$\frac{\partial}{\partial t}(\phi S_{\mathrm{w}}\rho_{\mathrm{w}})+\nabla\cdot(\chi_{\mathrm{w}}\rho_{\mathrm{w}}\mathbf{v}_{\mathrm{w}})=-\dot{m}\text{ (Sink).}\tag{7}$$

The moisture content of the binary mixture is determined by using equations (3) in equation (6) and separating the resulting equation for air and steam (there are no internal sources for air) to give

$$\frac{\partial}{\partial t}(\phi S_{\mathrm{gv}}\rho_{\mathrm{g}})+\nabla\cdot(\chi_{\mathrm{gv}}\rho_{\mathrm{g}}\mathbf{v}_{\mathrm{gv}})=\dot{m}\tag{8}$$

$$\frac{\partial}{\partial t}(\phi S_{\mathrm{ga}}\rho_{\mathrm{g}})+\nabla\cdot(\chi_{\mathrm{ga}}\rho_{\mathrm{g}}\mathbf{v}_{\mathrm{ga}})=0.\tag{9}$$

Product differentiation of equation (5) and substitution of equations (7)–(9) gives the energy equation as

$$\begin{aligned}&\phi S_{\mathrm{gv}}\rho_{\mathrm{g}}\frac{\partial h_{\mathrm{gv}}}{\partial t}+\phi S_{\mathrm{ga}}\rho_{\mathrm{g}}\frac{\partial h_{\mathrm{ga}}}{\partial t}+\phi S_{\mathrm{w}}\rho_{\mathrm{w}}\frac{\partial h_{\mathrm{w}}}{\partial t}\\&\quad+(1-\phi)\rho_{\mathrm{s}}\frac{\partial h_{\mathrm{s}}}{\partial t}+h_{\mathrm{vap}}\dot{m}+\chi_{\mathrm{gv}}\rho_{\mathrm{g}}\mathbf{v}_{\mathrm{gv}}\cdot\nabla h_{\mathrm{gv}}\\&\quad+\chi_{\mathrm{ga}}\rho_{\mathrm{g}}\mathbf{v}_{\mathrm{ga}}\cdot\nabla h_{\mathrm{ga}}+\chi_{\mathrm{w}}\rho_{\mathrm{w}}\mathbf{v}_{\mathrm{w}}\cdot\nabla h_{\mathrm{w}}\\&=\nabla\cdot((k_{\mathrm{g}}\chi_{\mathrm{g}}+k_{\mathrm{w}}\chi_{\mathrm{w}}+k_{\mathrm{g}}(1-\chi))\nabla T)\end{aligned}\tag{10}$$

where $h_{\mathrm{vap}}=h_{\mathrm{gv}}-h_{\mathrm{w}}$ is the latent heat of evaporation.

The basic equations, equations (6)–(8) and (10), are augmented by the following relations:

(i) Fick's law for diffusion [27]

$$\begin{aligned}\chi_{\mathrm{gv}}\mathbf{v}_{\mathrm{gv}}=\chi_{\mathrm{gv}}\mathbf{v}_{\mathrm{g}}-\chi_{\mathrm{g}}D\nabla\left(\frac{S_{\mathrm{gv}}}{S_{\mathrm{g}}}\right)&=\chi_{\mathrm{gv}}\mathbf{v}_{\mathrm{g}}\\&\quad-\chi_{\mathrm{g}}D\frac{M_{\mathrm{a}}M_{\mathrm{v}}}{M^2}\nabla\left(\frac{P_{\mathrm{gv}}}{P_{\mathrm{g}}}\right);\end{aligned}$$

(ii) Darcy's law for momentum [17, 20]

$$\chi_{\mathrm{g}}\mathbf{v}_{\mathrm{g}}=-\frac{KK_{\mathrm{g}}}{\mu_{\mathrm{g}}}\nabla P_{\mathrm{g}},\quad \chi_{\mathrm{w}}\mathbf{v}_{\mathrm{w}}=-\frac{KK_{\mathrm{w}}}{\mu_{\mathrm{w}}}(\nabla P_{\mathrm{w}}-\rho_{\mathrm{w}}\mathbf{g});$$

(iii) thermodynamic relations

$$P_{\mathrm{g}}=\frac{\rho_{\mathrm{g}}RT}{M},\quad P_{\mathrm{gv}}=\frac{\rho_{\mathrm{gv}}RT}{M_{\mathrm{v}}},\quad \rho_{\mathrm{gv}}=\frac{S_{\mathrm{gv}}\rho_{\mathrm{g}}}{S_{\mathrm{g}}},$$

$$M=M_{\mathrm{a}}-(M_{\mathrm{a}}-M_{\mathrm{v}})\frac{P_{\mathrm{gv}}}{P_{\mathrm{g}}};$$

(iv) Clausius–Clapeyron equation for the wet region [17]

$$P_{\mathrm{gv}}=P_{\mathrm{gv1}}\exp\left(-\frac{M_{\mathrm{v}}h_{\mathrm{vap}}}{R}\left(\frac{1}{T}-\frac{1}{T_1}\right)\right);$$

(v) capillary pressure [26]

$$\begin{aligned}P_{\mathrm{c}}(S,T)=P_{\mathrm{g}}-P_{\mathrm{w}}=\sqrt{\left(\frac{\phi}{K}\right)}\sigma(T)J(S),&\\ \sigma(T)=\sigma_0-\beta T,&\\ J(S)=0.364(1-\exp(-40(1-S)))&\\ +0.221(1-S)+\frac{0.005}{(S-S_{\mathrm{ir}})};&\end{aligned}$$

(vi) enthalpy–temperature relation

$$h=C_pT+\text{(some reference constant)};$$

(vii) relative permeabilities [16, 20]

$$
\begin{aligned}
K_{\mathrm{w}} &= \left(\frac{S-S_{\mathrm{ir}}}{1-S_{\mathrm{ir}}}\right)^3, && S > S_{\mathrm{ir}} \\
K_{\mathrm{w}} &= 0, && S \leqslant S_{\mathrm{ir}} \\
K_{\mathrm{g}} &= 1-1.11S, && S < S_{\mathrm{cr}} \\
K_{\mathrm{g}} &= 0, && S \geqslant S_{\mathrm{cr}}.
\end{aligned}
$$

Finally the boundary conditions must be specified.

On the adiabatic and impervious surface the fluxes of mass and energy are zero:

flux of air

$$\chi_{\mathrm{ga}}\rho_{\mathrm{g}}\mathbf{v}_{\mathrm{ga}}\cdot\mathbf{n} = 0; \tag{11}$$

flux of water

$$(\chi_{\mathrm{gv}}\rho_{\mathrm{g}}\mathbf{v}_{\mathrm{gv}} + \chi_{\mathrm{w}}\rho_{\mathrm{w}}\mathbf{v}_{\mathrm{w}})\cdot\mathbf{n} = 0; \tag{12}$$

flux of heat

$$((k_{\mathrm{g}}\chi_{\mathrm{g}} + k_{\mathrm{w}}\chi_{\mathrm{w}} + k_{\mathrm{s}}(1-\chi))\nabla T + \chi_{\mathrm{w}}\rho_{\mathrm{w}}h_{\mathrm{vap}}\mathbf{v}_{\mathrm{w}})\cdot\mathbf{n} = 0. \tag{13}$$

On the exposed surface the fluxes are continuous:

flux of water

$$(\chi_{\mathrm{gv}}\rho_{\mathrm{g}}\mathbf{v}_{\mathrm{gv}} + \chi_{\mathrm{w}}\rho_{\mathrm{w}}\mathbf{v}_{\mathrm{w}})\cdot\mathbf{n} = k_{\mathrm{m}}(\rho_{\mathrm{gv}} - \rho_{\mathrm{gv0}}); \tag{14}$$

flux of heat

$$((k_{\mathrm{g}}\chi_{\mathrm{g}} + k_{\mathrm{w}}\chi_{\mathrm{w}} + k_{\mathrm{s}}(1-\chi))\nabla T + \chi_{\mathrm{w}}\rho_{\mathrm{w}}h_{\mathrm{vap}}\mathbf{v}_{\mathrm{w}})\cdot\mathbf{n} = Q(T_0 - T); \tag{15}$$

pressure

$$P_{\mathrm{g}} = P_0. \tag{16}$$

Here Q is the heat transfer coefficient for unit area of the exposed face of the slab. Although it is temperature dependent [8], in this work it is taken as constant [20]. It should be noted that equation (14) is true if $P_{\mathrm{gv}} > P_{\mathrm{gv0}}$, otherwise we use the boundary condition $P_{\mathrm{gv}} = P_{\mathrm{gv0}}$.

At the evaporation front a proper treatment would require specification of the jump conditions, this will be done at a later date. For numerical simulations, it is sufficient to demand continuity of the relevant quantities across the evaporating front.

3. ONE-DIMENSIONAL EQUATIONS

For one-dimensional flow it is reasonable to assume that the surface saturation porosity χ is equal to the volume porosity ϕ and that the surface saturation S' is equal to the volume saturation S for gas and water, i.e.

$$\chi_{\mathrm{g}} = S'_{\mathrm{g}}\chi = S_{\mathrm{g}}\phi, \quad \chi_{\mathrm{gv}} = \phi S_{\mathrm{gv}},$$

$$\chi_{\mathrm{w}} = \phi S_{\mathrm{w}} \quad \text{and} \quad (1-\chi) = (1-\phi).$$

Also $S_{\mathrm{g}} = 1 - S_{\mathrm{w}}$. Information in the previous section and mathematical manipulation yields the following system of equations for the wet and dry regions.

3.1. *Wet region: variables $S = S_{\mathrm{w}}$, P_{g} and T*

The relation $P_{\mathrm{gv}} = P_{\mathrm{gv}}(T)$ allows the elimination of $\dot{m}$ from equations (6)–(8) and (10) to give

$$\left(1 - \frac{P_{\mathrm{gv}}M_{\mathrm{v}}}{R\rho_{\mathrm{w}}T}\right)\frac{\partial S}{\partial t} + \left(\frac{(1-S)(h_{\mathrm{vap}}M_{\mathrm{v}} - RT)M_{\mathrm{v}}P_{\mathrm{gv}}}{\rho_{\mathrm{w}}R^2T^3}\right)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[K_S\frac{\partial S}{\partial x} + K_{T1}\frac{\partial T}{\partial x} + K_{P1}\frac{\partial P_{\mathrm{g}}}{\partial x} + K_{\mathrm{gr}}\right] \tag{17}$$

$$
\begin{aligned}
&\left(\frac{\phi C_{p\mathrm{g}}(1-S)MP_{\mathrm{g}}}{RT} + \phi C_{p\mathrm{w}}\rho_{\mathrm{w}}S + (1-\phi)\rho_{\mathrm{s}}C_{p\mathrm{s}}\right)\frac{\partial T}{\partial t} \\
&\quad - \phi\rho_{\mathrm{w}}h_{\mathrm{vap}}\frac{\partial S}{\partial t} = \frac{\partial}{\partial x}\left(K_{\mathrm{e}}\frac{\partial T}{\partial x}\right) \\
&\quad - \phi h_{\mathrm{vap}}\rho_{\mathrm{w}}\frac{\partial}{\partial x}\left(K_S\frac{\partial S}{\partial x} + K_T\frac{\partial T}{\partial x} + K_P\frac{\partial P_{\mathrm{g}}}{\partial x} + K_{\mathrm{gr}}\right) \\
&\quad + \left[\phi\rho_{\mathrm{w}}C_{p\mathrm{w}}\left(K_S\frac{\partial S}{\partial x} + K_T\frac{\partial T}{\partial x} + K_{\mathrm{gr}}\right)\right. \\
&\qquad \left. + \left(\phi\rho_{\mathrm{w}}C_{p\mathrm{w}}K_P + \frac{C_{p\mathrm{g}}KK_{\mathrm{g}}MP_{\mathrm{g}}}{\mu_{\mathrm{g}}RT}\right)\frac{\partial P_{\mathrm{g}}}{\partial x}\right]\frac{\partial T}{\partial x}
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
&\left(1 - \frac{MP_{\mathrm{g}}}{R\rho_{\mathrm{w}}T}\right)\frac{\partial S}{\partial t} + \frac{M_{\mathrm{a}}(1-S)}{R\rho_{\mathrm{w}}T}\frac{\partial P_{\mathrm{g}}}{\partial t} \\
&\quad + \left(\frac{(1-S)[(M_{\mathrm{v}} - M_{\mathrm{a}})M_{\mathrm{v}}h_{\mathrm{vap}}P_{\mathrm{gv}} - P_{\mathrm{g}}MRT]}{\rho_{\mathrm{w}}R^2T^3}\right)\frac{\partial T}{\partial t} \\
&\quad = \frac{\partial}{\partial x}\left(K_S\frac{\partial S}{\partial x} + K_T\frac{\partial T}{\partial x} + K_{P2}\frac{\partial P_{\mathrm{g}}}{\partial x} + K_{\mathrm{gr}}\right)
\end{aligned} \tag{19}
$$

where the coefficients are defined as

$$K_S = -\frac{KK_{\mathrm{w}}}{\phi\mu_{\mathrm{w}}}\frac{\partial P_{\mathrm{c}}}{\partial S}; \quad K_{\mathrm{gr}} = \frac{\rho_{\mathrm{w}}KK_{\mathrm{w}}g}{\phi\mu_{\mathrm{w}}};$$

$$K_{\mathrm{e}} = \phi k_{\mathrm{g}} + (1-\phi)k_{\mathrm{s}} + \phi(k_{\mathrm{w}} - k_{\mathrm{g}})S; \quad K_T = -\frac{KK_{\mathrm{w}}}{\phi\mu_{\mathrm{w}}}\frac{\partial P_{\mathrm{c}}}{\partial T};$$

$$K_{T1} = K_T + \frac{(1-S)DM_{\mathrm{a}}M_{\mathrm{v}}^2h_{\mathrm{vap}}P_{\mathrm{gv}}}{\rho_{\mathrm{w}}MR^2T^3}; \quad K_P = \frac{KK_{\mathrm{w}}}{\phi\mu_{\mathrm{w}}};$$

$$K_{P1} = K_P + \frac{KK_{\mathrm{g}}M_{\mathrm{v}}P_{\mathrm{gv}}}{\phi\mu_{\mathrm{g}}\rho_{\mathrm{w}}RT} - \frac{(1-S)DM_{\mathrm{a}}M_{\mathrm{v}}P_{\mathrm{gv}}}{\rho_{\mathrm{w}}RTMP_{\mathrm{g}}};$$

$$K_{P2} = K_P + \frac{KK_{\mathrm{g}}MP_{\mathrm{g}}}{\phi\mu_{\mathrm{g}}\rho_{\mathrm{w}}RT}.$$

3.1.1. *Boundary conditions.*

At $x = 0$:

$$K_S\frac{\partial S}{\partial x} + K_{T1}\frac{\partial T}{\partial x} + K_{P1}\frac{\partial P_{\mathrm{g}}}{\partial x} + K_{\mathrm{gr}} = 0 \tag{20a}$$

$$K_{\mathrm{e}}\frac{\partial T}{\partial x} - \phi\rho_{\mathrm{w}}h_{\mathrm{vap}}\left(K_S\frac{\partial S}{\partial x} + K_T\frac{\partial T}{\partial x} + K_P\frac{\partial P_{\mathrm{g}}}{\partial x} + K_{\mathrm{gr}}\right) = 0 \tag{20b}$$

$$(K_{P1}-K_{P2})\frac{\partial P_g}{\partial x}+(K_{T1}-K_T)\frac{\partial T}{\partial x}=0. \qquad (20c)$$

At $x = L$:

$$-\left(K_S\frac{\partial S}{\partial x}+K_{T1}\frac{\partial T}{\partial x}+K_{P1}\frac{\partial P_g}{\partial x}+K_{gr}\right) = \frac{k_m M_v}{R\phi\rho_w}\left(\frac{P_{gv}}{T}-\frac{P_{gv0}}{T_0}\right) \qquad (21a)$$

$$K_e\frac{\partial T}{\partial x}-\phi\rho_w h_{vap}\left(K_S\frac{\partial S}{\partial x}+K_T\frac{\partial T}{\partial x}+K_P\frac{\partial P_g}{\partial x}+K_{gr}\right) = Q(T_0-T) \qquad (21b)$$

$$P_g(L,t) = P_0. \qquad (21c)$$

3.1.2. *Initial conditions.*

$$T(x,0) = T_1 \qquad (22a)$$

$$P_g(x,0) = P_0 \qquad (22b)$$

$$\frac{\partial P_c}{\partial x} = \rho_w g. \qquad (22c)$$

3.2. *Dry region: variables are P_{gv}, P_g and T*

In this region $\dot{m} = 0$, S is taken as S_{ir} and gravity is negligible. Equations (6), (8) and (10) become

$$\frac{\phi M_v}{RT}\frac{\partial P_{gv}}{\partial t}-\frac{\phi M_v P_{gv}}{RT^2}\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(K_{v1}\frac{\partial P_{gv}}{\partial x}+K_{v2}\frac{\partial P_g}{\partial x}\right) \qquad (23)$$

$$\left(\frac{\phi M P_g C_{pg}}{RT}+(1-\phi)\rho_s C_{ps}\right)\frac{\partial T}{\partial t} = (\phi k_g+(1-\phi)k_s) \times\frac{\partial^2 T}{\partial x^2}+\frac{C_{pg}KK_g M P_g}{\mu_g RT}\frac{\partial P_g}{\partial x}\frac{\partial T}{\partial x} \qquad (24)$$

$$\frac{\phi(M_v-M_a)}{RT}\frac{\partial P_{gv}}{\partial t}-\frac{\phi M P_g}{RT^2}\frac{\partial T}{\partial t} +\frac{\phi M_a}{RT}\frac{\partial P_g}{\partial t} = \frac{\partial}{\partial x}\left(K_{v3}\frac{\partial P_g}{\partial x}\right) \qquad (25)$$

where the coefficients are defined as

$$K_{v1} = \frac{\phi M_a M_v D}{MRT};$$

$$K_{v2} = \frac{KK_g P_{gv} M_v}{\mu_g RT}-\frac{\phi M_v M_a D P_{gv}}{MRTP_g};$$

$$K_{v3} = \frac{KK_g M P_g}{\mu_g RT}.$$

3.2.1. *Boundary conditions.*

At $x = L$:

$$-\left(K_{v1}\frac{\partial P_{gv}}{\partial x}+K_{v2}\frac{\partial P_g}{\partial x}\right) = \frac{K_m M_v}{R}\left(\frac{P_{gv}}{T}-\frac{P_{gv0}}{T_0}\right); \quad P_{gv} > P_{gv0} \qquad (26a)$$

$$(\phi k_g+(1-\phi)k_s)\frac{\partial T}{\partial x} = Q(T_0-T) \qquad (26b)$$

$$P_g(L,t) = P_0. \qquad (26c)$$

3.2.2. *At evaporation front.*

$T(x,t)$ and $P_g(x,t)$ are continuous, $S = S_{ir}$ (27a)

and

$P_{gv} = P_{gv}(T_m)$ where T_m is the temperature at the front. (27b)

4. NON-DIMENSIONAL FORM OF EQUATIONS

The equations of the last section are transformed to non-dimensional form by using the scaling

$$x' = x/L; \quad \tau = \alpha\tau/L^2; \quad \Theta(x',\tau) = \frac{T(x,t)-T_1}{T_0-T_1};$$

$$\tilde{P}(x',\tau) = \frac{P_g(x,t)}{P_0}; \quad \tilde{P}_{gv}(x',\tau) = \frac{P_{gv}(x,t)}{P_0}$$

and the non-dimensional parameters defined in the Appendix.

4.1. *Wet region*

The variables are S, Θ, $\tilde{P}$, written in vector form as $\mathbf{u}_w = (S, \Theta, \tilde{P})^T$. Equations (17)–(22) become

$$\mathbf{A}_w\frac{\partial \mathbf{u}_w}{\partial \tau} = \frac{\partial}{\partial x'}\left(\mathbf{D}_w\frac{\partial \mathbf{u}_w}{\partial x}+\mathbf{d}_w\right)+\mathbf{a}_w. \qquad (28)$$

4.1.1. *Boundary conditions.*

$$x' = 0: \quad \mathbf{B}_w\frac{\partial \mathbf{u}_w}{\partial x'} = \mathbf{b}_w \qquad (29)$$

$$x' = 1: \quad \mathbf{C}_w\frac{\partial \mathbf{u}_w}{\partial x'} = \mathbf{c}_w. \qquad (30)$$

4.1.2. *Initial conditions.*

$$\tau = 0: \quad \mathbf{u}_w(x',0) = (S(x',0),0,1)^T \qquad (31a)$$

where

$S(x',0)$ is defined by $J(S) = J\{S(0,0)\}+\sigma_2 x'$. (31b)

The coefficients of the matrices $\mathbf{A}_w(\mathbf{u}_w)$, $\mathbf{B}_w(\mathbf{u}_w)$, $\mathbf{C}_w(\mathbf{u}_w)$, $\mathbf{D}_w(\mathbf{u}_w)$ and of the vectors $\mathbf{a}_w$, $\mathbf{b}_w$, $\mathbf{c}_w$, $\mathbf{d}_w$ are defined in the Appendix.

4.2. *Dry region*

The variables are $\tilde{P}_{gv}$, Θ, $\tilde{P}$, written in vector form as $\mathbf{u}_d = (\tilde{P}_{gv}, \Theta, \tilde{P})^T$. Equations (23)–(27) become

$$\mathbf{A}_d\frac{\partial \mathbf{u}_d}{\partial \tau} = \frac{\partial}{\partial x'}\left(\mathbf{D}_d\frac{\partial \mathbf{u}_d}{\partial x'}\right)+\mathbf{a}_d. \qquad (32)$$

4.2.1. *Boundary conditions.*

$$x' = 1: \quad \mathbf{C}_d \frac{\partial \mathbf{u}_d}{\partial x'} = \mathbf{c}_d. \tag{33a}$$

4.2.2. *Evaporation front.*

$S = S_{ir}$, Θ and $\tilde{P}$

$$\text{are continuous and } \tilde{P}_{gv} = \tilde{P}_{vl} F(\Theta). \tag{33b}$$

The coefficients of the matrices $\mathbf{A}_d(\mathbf{u}_d)$, $\mathbf{C}_d(\mathbf{u}_d)$, $\mathbf{D}_d(\mathbf{u}_d)$ and of the vectors $\mathbf{a}_d$, $\mathbf{c}_d$ are defined in the Appendix.

5. FINITE DIFFERENCE SOLUTION

The numerical solution technique employed to solve the system of three non-linear partial differential equations is the fully implicit time discretization scheme which is known to yield stable solutions for any selection of the space and time grid spacings. Details of the numerical procedure and its validity can be found in ref. [28]; finite difference approximations of moving boundary problems are discussed in ref. [29]. Linearization was always accomplished by evaluating the non-linear coefficients at the previous time step [17].

The (x', τ) domain is subdivided by using

$$x'_i = ih, \quad \tau_j = j\Delta\tau \quad \text{where} \quad i = 1, 2, \ldots, (3N-1); \quad j = 1, 2, \ldots, \tau_{\max}.$$

Upwind differencing was implemented for the convection terms in the energy equation and central differencing for the other terms. The system of algebraic equations reduces to the form

$$\mathbf{U}^{j+1} = (\mathbf{I} - \Delta\tau \mathbf{P}(\mathbf{U}^j)^{-1}\mathbf{R}(\mathbf{U}^j))^{-1} \times [\mathbf{U}^j + \Delta\tau \mathbf{P}(\mathbf{U}^j)^{-1}\mathbf{Q}(\mathbf{U}^j)] \tag{34}$$

where

$$\mathbf{U} = (u_1, \Theta_1, \tilde{P}_1, u_2, \Theta_2, \tilde{P}_2, \ldots, u_i, \Theta_i, \tilde{P}_i, \ldots, u_N, \Theta_N)^{\mathrm{T}} \tag{35}$$

and

$$u_i = \begin{cases} S_i; & i < I \\ 0; & i = I \\ \tilde{P}_{gvi}; & i > I \end{cases}$$

with

$$I = \min\{i \mid S_i \leqslant S_{ir}\}.$$

The matrix $\mathbf{P}(\mathbf{U}) = \operatorname{diag}(\mathbf{P}_1, \ldots, \mathbf{P}_i, \ldots, \mathbf{P}_N)$ and

$$\mathbf{P}_i = \begin{cases} \mathbf{A}_{wi}; & i < I \\ \mathbf{A}_I; & i = I \\ \mathbf{A}_{di}; & i > I \end{cases}$$

with $\mathbf{A}_{wi} = \mathbf{A}_w(\mathbf{U}_{wi})$, $\mathbf{A}_{di} = \mathbf{A}_d(\mathbf{U}_{di})$ as the matrices given in the last section, $\mathbf{P}_N$ and $\mathbf{A}_I$ are basically 2×2 matrices.

Similarly $\mathbf{R}(\mathbf{U})$ is a tridiagonal block matrix involving $\mathbf{D}_w(\mathbf{U}_{wi})$ and $\mathbf{D}_d(\mathbf{U}_{di})$ for $i < I$ and $i > I$, respectively, and $\mathbf{Q}(\mathbf{U})$ is a vector involving $\mathbf{a}_w(\mathbf{U}_{wi})$ and $\mathbf{a}_d(\mathbf{U}_{di})$ for $i < I$ and $i > I$.

Initially:

$$\mathbf{U} = (S_1, 0, 1, S_2, 0, 1, \ldots, S_N, 0)^{\mathrm{T}}. \tag{36}$$

An efficient solution technique which solved the system of algebraic equations (34) employed full row and column pivoting. Stability was assured by use of the fully implicit scheme (the Crank–Nicolson method was found to be unstable).

A predictor–corrector method developed via the Adams method was also implemented on the fully non-linear system. It was found that this method required extremely small time steps for convergence. Consequently it was very time consuming during processing.

The fully implicit scheme agreed well with results produced by the predictor–corrector scheme and has the advantage of faster processing times.

6. GRAPHICAL RESULTS AND DISCUSSION

The graphical results presented in Figs. 2–9 are for the porous material brick. The physical data for brick is well documented in refs. [8, 11, 20]. These values together with the actual physical constants for air and water and the controlling constants:

$$T_0 = 360.15 \text{ K}; \quad T_1 = 320.15 \text{ K}$$

$$P_0 = 1.0133 \times 10^5 \text{ [Pa]}; \quad P_{gv0} = 2.55 \times 10^3 \text{ [Pa]}$$

$$P_{gv1} = 8.5 \times 10^3 \text{ [Pa]}; \quad L = 0.01 \text{ m}$$

$$S_{ir} = 0.09; \quad S_{cr} = 1/1.1$$

were used in all numerical calculations.

It should be noted that most of the physical constants do really vary with temperature, but the variation is small for the working interval.

In accordance with physical expectations, at the start of drying, the temperature of the porous medium falls to the wet bulb temperature (Fig. 5). During the 'constant drying rate' period the temperature (Fig. 5) and the pressure (Fig. 7) are uniform and almost

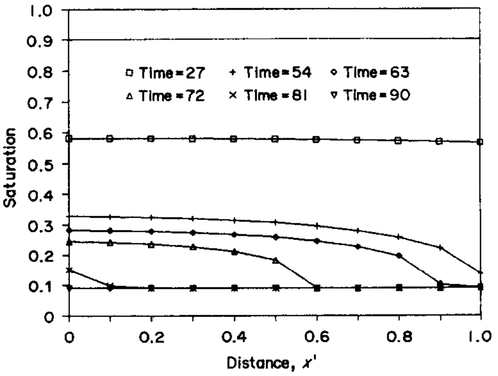

FIG. 2. Moisture content as a function of distance at various times (all nondimensional).

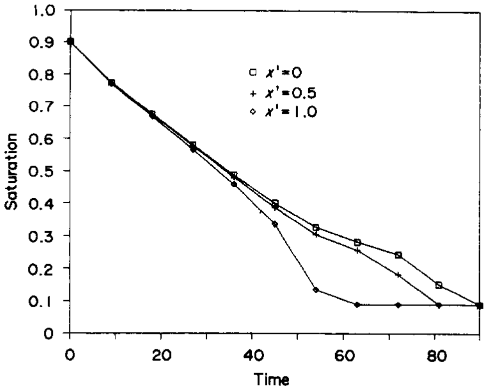

Fig. 3. Moisture content profile in time at various distances (all nondimensional).

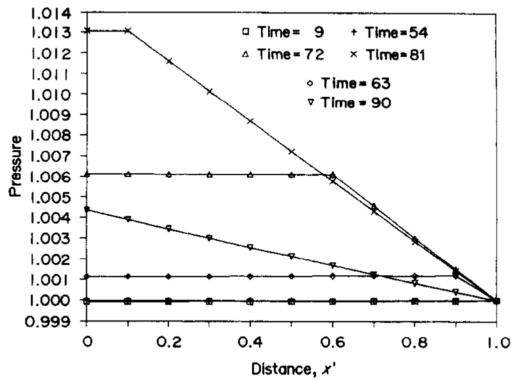

Fig. 6. Pressure distance profiles at various times (all nondimensional).

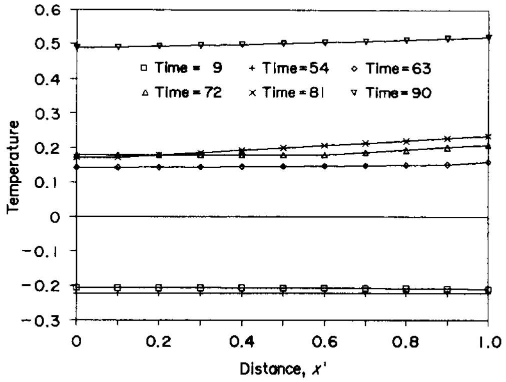

Fig. 4. Temperature as a function of distance at various times (all nondimensional).

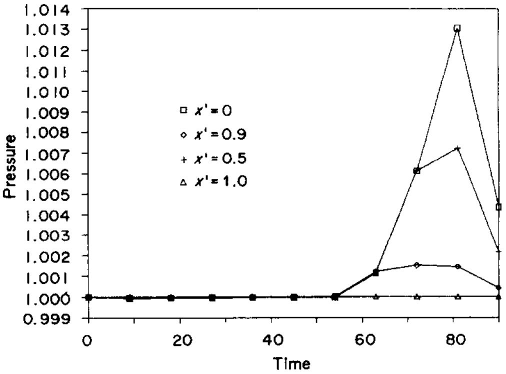

Fig. 7. Pressure vs time at various distances (all nondimensional).

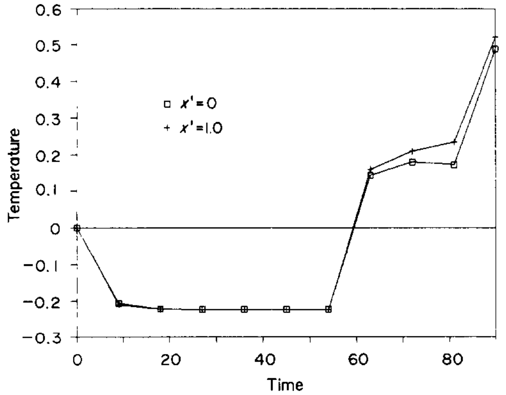

Fig. 5. Temperature profile in times at various distances (all nondimensional).

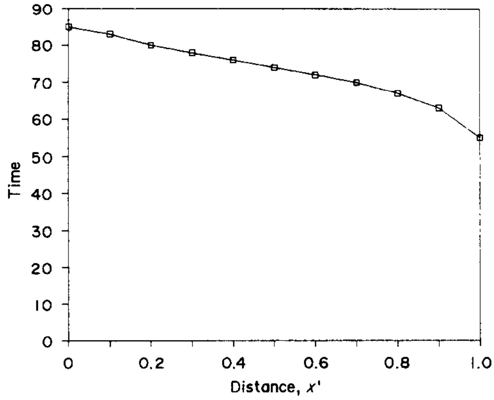

Fig. 8. Position of evaporation front.

constant. In fact the model is well approximated by the constant temperature and pressure model (see Section 7). The time taken for the saturation, S, at the exposed surface, to fall from 0.9 to S_{ir} (Fig. 3), is a non-dimensional time of 55, which in actual time is 91 min. This seems slightly longer than the results in ref. [20] but see the comment on evaporation front later. The corresponding temperature at the exposed surface is 45°C and the saturation value at the impervious surface is 0.315.

When the moisture content reaches S_{ir} at the exposed surface there is a slowing down of the drying process. As soon as the dry zone appears (Fig. 2), the temperature starts increasing (Fig. 5), as does the pressure (Fig. 7), which is at the maximum at the evaporation front. The front is marked by the discontinuity in temperature (Fig. 4) and pressure (Fig. 6) gradients. This would be accounted for by jump conditions across the front, involving latent heat of evaporation as well as the change in coefficients in our

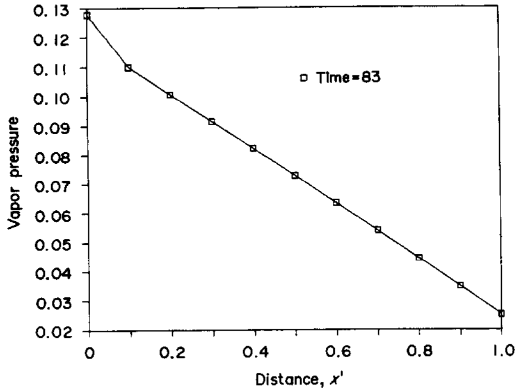

FIG. 9. Vapour pressure vs distance (both nondimensional), when the front reaches impervious face.

equations. A casual glance at Fig. 4, which was drawn by computer, may indicate no discontinuity in slope, but a more thorough examination will reveal a jump in slope where the front is.

The position of the front, which moves from the exposed surface to the impervious surface, is plotted in Fig. 8 which exhibits two distinct behaviours: one for small time and one for large time. The shape for large time is the usual $t^{1/2}$ shape found in other moving boundary value problems, see ref. [23] and references cited therein. The shape is similar to Luikov's curve of evaporation surface deepening (Fig. 9.36(c) of ref. [12]) for quartz sand. It is to be noted that Luikov's curve is not brought all the way to the exposed surface. There may be a period of time when it is hard to determine experimentally whether the front has left the exposed surface. The gap requires much more thorough mathematical analysis for small time which will be done at a later date. This may also explain why our time for the front to appear at the exposed surface is different from that of ref. [20]. The total time taken to dry the brick, i.e. when the front reaches the impervious surface is a non-dimensional time of 83, which in actual time is 140 min. This compares well with ref. [20]. The corresponding temperatures at $x = 0$ and L are 53 and 55°C, respectively. Figure 9 shows the water vapour pressure profile as the front reaches the impervious face.

Most experimental methods and results in drying are concerned with determination of drying kinetic curves, since a knowledge of the drying kinetics is necessary for drier design. Drying kinetics are connected with the changes of average material moisture content and average material temperature with time, contrary to drying dynamics which describes changes in the temperature, pressure and moisture profiles throughout the drying body. Consequently experimental results involving drying dynamics that could be compared directly with our results are very few. Harmathy [8] gives a few temperature curves for brick but with different drying agent characteristics from ours; however his T_i curve for Experiment 3 (his Fig. 6) has the same shape as our temperature curves in Fig. 5. Luikov [12] gives a few moisture content and temperature distribution curves (his Figs. 5.18(b), 9.36(a) and 9.37) obtained experimentally for sand and with different drying agent characteristics from ours: the shapes of the curves are similar to our Figs. 2–4.

7. SIMPLIFIED MODELS

Two models which are often used in the literature are the constant pressure model and the constant temperature and pressure model. We briefly exhibit how these models can be recovered from our model and discuss the parameters involved.

7.1. *Simple model for moisture distribution* [16, 18, 19]

If we set $\Theta = 0$ and $\tilde{P} = 1$ for $0 \leqslant x' \leqslant 1$, equations (28)–(31) reduce to a single equation

$$\frac{\partial S}{\partial \tau} = \frac{\partial}{\partial x'}\left(\tilde{K}_S \frac{\partial S}{\partial x'} + \tilde{K}_{\mathrm{gr}}\right).$$

7.1.1. *Boundary conditions.*

$$x' = 0: \quad \tilde{K}_S \frac{\partial S}{\partial x'} + \tilde{K}_{\mathrm{gr}} = 0$$

$$x' = 1: \quad \tilde{K}_S \frac{\partial S}{\partial x'} + \tilde{K}_{\mathrm{gr}} = K_{\mathrm{mw}}\left(\frac{P_{\mathrm{s0}}}{\Theta_0} - \frac{1}{\Theta_1}\right).$$

7.1.2. *Initial conditions.*

$$J(S) = J(S_0) + \sigma_2 x'$$

provided:

(i) $$a_{\mathrm{s}} = \lambda_{\mathrm{v}} K_{\mathrm{mw}}\left(\frac{1}{\Theta_1} - \frac{P_{\mathrm{s0}}}{\Theta_0}\right);$$

that is, the system is at the wet bulb temperature, T_1;

(ii) the parameters v_{g} and v_{gs} are negligible; v_{g} is effectively the ratio of air density to water density and v_{gs} is the ratio of steam density to water density.

7.2. *Constant pressure model* [17]

If we set $\tilde{P} = 1$ for $0 \leqslant x' \leqslant 1$ equations (28)–(31) reduce to the following two equations:

$$\frac{\partial S}{\partial \tau} = \frac{\partial}{\partial x'}\left(\tilde{K}_S \frac{\partial S}{\partial x'} + \tilde{K}_T \frac{\partial \Theta}{\partial x'} + \tilde{K}_{\mathrm{gr}}\right)$$

$$(1 + a_{\mathrm{w}} S)\frac{\partial \Theta}{\partial \tau} = \frac{\partial}{\partial x'}\left(\tilde{K}_{\mathrm{e}} \frac{\partial \Theta}{\partial x'}\right) + a_{\mathrm{w}}\left(\tilde{K}_S \frac{\partial S}{\partial x'} + \tilde{K}_T \frac{\partial \Theta}{\partial x'} + \tilde{K}_{\mathrm{gr}}\right)\frac{\partial \Theta}{\partial x'}.$$

7.2.1. *Boundary conditions.*

$$x' = 0: \quad \tilde{K}_S \frac{\partial S}{\partial x'} + \tilde{K}_{\mathrm{gr}} = 0, \quad \frac{\partial \Theta}{\partial x'} = 0;$$

$$x' = 1: \quad \tilde{K}_S \frac{\partial S}{\partial x'} + \tilde{K}_T \frac{\partial \Theta}{\partial x'} + \tilde{K}_{\mathrm{gr}} = K_{\mathrm{mw}} \left(\frac{P_{\mathrm{s0}}}{\Theta_0} - \frac{F(\Theta}{(\Theta + \Theta_1)} \right),$$

$$\tilde{K}_{\mathrm{c}} \frac{\partial \Theta}{\partial x'} = a_{\mathrm{s}}(1 - \Theta) + \lambda_{\mathrm{v}} K_{\mathrm{mw}} \left(\frac{P_{\mathrm{s0}}}{\Theta_0} - \frac{F(\Theta)}{(\Theta + \Theta_1)} \right).$$

7.2.2. *Initial conditions.*

$$\Theta = 0; \quad J(S) = J(S_0) + \sigma_2 x'$$

provided the parameters v_{g} and v_{gs} are negligible.

8. CONCLUSION

The drying model formulated in this study has the form of the Luikov equations and is in the spirit of Whitaker's work but has the advantage that all coefficients are constructed from well-known experimental results. The results obtained in this work for the drying of brick agree with available experimental results and the results in ref. [20] based on Whitaker's equations.

The paper is written for specialists in the area of mathematical modelling of drying processes but the computer program which is parameter driven can be easily understood and used by workers in this area. In particular dryer designers can use the program with different input parameters to see which values give the optimal results. Ideally one should solve the three-dimensional case including the geometry of the drier and the boundary layer equations for air flow over the exposed surface. This data forms the basis for the design of the remaining components of the drier installation [30,31].

As described in the introduction, the formulation in this paper can be extended to include 'non-equilibrium' processes as well as problems where stress development is of interest.

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APPENDIX

Non-dimensional parameters

$$\Delta T = (T_0 - T_1)\,; \quad \Theta_0 = \frac{T_0}{\Delta T}\,; \quad \Theta_1 = \frac{T_1}{\Delta T}\,;$$

$$P_{s0} = \frac{P_{gv0}}{P_{gv1}}\,; \quad P_{s1} = \frac{P_{gv0}}{P_0}\,; \quad P_{v1} = \frac{P_{gv1}}{P_0}\,;$$

$$\alpha = \frac{k_s(1-\phi)+k_g\phi}{(1-\phi)\rho_s C_{ps}}\,; \quad \sigma_1 = \frac{\beta \Delta T}{\sigma_0}\,; \quad \nu_{gs} = \frac{P_{gv1} M_v}{R\rho_w \Delta T}\,;$$

$$\nu_g = \frac{P_0 M_a}{R\rho_w \Delta T}\,; \quad \lambda_{vs} = \frac{h_{vap} M_v}{R \Delta T}\,; \quad \lambda_v = \frac{\phi \rho_w h_{vap}}{\Delta T(1-\phi)\rho_s C_{ps}}\,;$$

$$a_w = \frac{\phi C_{pw}\rho_w}{(1-\phi)\rho_s C_{ps}}\,; \quad a_g = \frac{\phi C_{pg}\rho_w}{(1-\phi)\rho_s C_{ps}}\,;$$

$$\kappa_g = \frac{\phi(k_w - k_g)}{\phi k_g + (1-\phi)k_s}\,; \quad \gamma_{gs} = \frac{M_v - M_a}{M_a}\,; \quad \gamma_g = \frac{(M_v - M_a)P_{gv1}}{M_a P_0}\,;$$

$$a_s = \frac{QL}{\phi k_g + (1-\phi)k_s}\,; \quad K_{mw} = \frac{k_m P_{gv1} L M_v}{\alpha \rho_w \phi R \Delta T}\,;$$

$$\delta_s = \sqrt{\left(\frac{K}{\phi}\right)}\frac{\sigma_0}{\mu_w \alpha}\,; \quad \delta_T = \sqrt{\left(\frac{K}{\phi}\right)}\frac{\sigma_0 \sigma_1}{\mu_w \alpha}\,; \quad \delta_P = \frac{KP_0}{\phi \mu_w \alpha}\,;$$

$$\delta_v = \frac{KP_0}{\phi \mu_g \alpha}\,; \quad \delta_{gr} = \frac{\rho_w K L g}{\phi \mu_w \alpha}\,; \quad \tilde{D} = \frac{D}{\alpha}\,;$$

$$\sigma_2 = \sqrt{\left(\frac{K}{\phi}\right)}\frac{\rho_w g L}{\sigma_0(1-\sigma_1\Theta_1)}\,.$$

Wet region.

$$F(\Theta) = \exp\left(\frac{\lambda_{vs}\Theta}{\Theta_1(\Theta+\Theta_1)}\right); \quad \tilde{\sigma}(\Theta) = 1 - \sigma_1(\Theta+\Theta_1)\,;$$

$$\tilde{M}_w = 1 + \frac{\gamma_g F(\Theta)}{\tilde{P}}\,; \quad \tilde{K}_S = -\delta_S K_w(S)\tilde{\sigma}(\Theta)J'(S)\,;$$

$$\tilde{K}_{gr} = \delta_{gr} K_w(S)\,; \quad \tilde{K}_v = \delta_v K_g(S)\,;$$

$$\tilde{K}_T = \delta_T K_w(S) J(S)\,; \quad \tilde{K}_{T1} = \tilde{K}_T + \frac{\tilde{D}\lambda_{vs}\nu_{gs}(1-S)F(\Theta)}{\tilde{M}_w(\Theta+\Theta_1)^3}\,;$$

$$\tilde{K}_e = 1 + \kappa_g S\,; \quad \tilde{K}_P = \delta_P K_w(S)\,;$$

$$\tilde{K}_{P1} = \tilde{K}_P + \frac{\nu_{gs}\tilde{K}_v F(\Theta)}{(\Theta+\Theta_1)} - \frac{\tilde{D}\nu_{gs}(1-S)F(\Theta)}{\tilde{M}_w \tilde{P}(\Theta+\Theta_1)}\,;$$

$$\tilde{K}_{P2} = \tilde{K}_P + \frac{\nu_g \tilde{K}_v \tilde{M}_w \tilde{P}}{(\Theta+\Theta_1)}\,.$$

Dry region.

$$\tilde{M}_d = \frac{\gamma_{gs}\tilde{P}_{gv}}{\tilde{P}}\,; \quad \tilde{K}_{gv1} = \frac{\tilde{D}}{\tilde{M}_d(\Theta+\Theta_1)}\,;$$

$$\tilde{K}_{gv2} = \left(\tilde{K}_v - \frac{\tilde{D}}{\tilde{M}_d \tilde{P}}\right)\frac{\tilde{P}_{gv}}{(\Theta+\Theta_1)}\,; \quad \tilde{K}_{gv3} = \frac{\tilde{K}_v \tilde{M}_d \tilde{P}}{(\Theta+\Theta_1)}\,.$$

Matrix and vector coefficients
Wet region.

$$A_w(u_w) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$a_{11} = 1 - \frac{\nu_{gs}F(\Theta)}{(\Theta+\Theta_1)}\,;$$

$$a_{12} = \frac{(1-S)\nu_{gs}F(\Theta)}{(\Theta+\Theta_1)^2}\left(\frac{\lambda_{vs}}{(\Theta+\Theta_1)} - 1\right);$$

$$a_{13} = 0\,;$$

$$a_{21} = -\lambda_v\,;$$

$$a_{22} = 1 + a_w S + \frac{a_g \nu_g \tilde{M}_w \tilde{P}(1-S)}{(\Theta+\Theta_1)}\,;$$

$$a_{23} = 0\,;$$

$$a_{31} = 1 - \frac{\nu_g \tilde{M}_w \tilde{P}}{(\Theta+\Theta_1)}\,;$$

$$a_{32} = \frac{\nu_g(1-S)[\lambda_{vs}\gamma_g F(\Theta) - \tilde{M}_w \tilde{P}(\Theta+\Theta_1)]}{(\Theta+\Theta_1)^3}\,;$$

$$a_{33} = \frac{\nu_g(1-S)}{(\Theta+\Theta_1)}\,;$$

$$\mathbf{D}_w(\mathbf{u}_w) = \begin{pmatrix} \tilde{K}_S & \tilde{K}_{T1} & \tilde{K}_{P1} \\ -\lambda_v \tilde{K}_S & (\tilde{K}_e - \lambda_v \tilde{K}_T) & -\lambda_v \tilde{K}_P \\ \tilde{K}_S & \tilde{K}_T & \tilde{K}_{P2} \end{pmatrix};$$

$$\mathbf{d}_w = \begin{pmatrix} \tilde{K}_{gr} \\ -\lambda_v \tilde{K}_{gr} \\ \tilde{K}_{gr} \end{pmatrix};$$

$$\mathbf{a}_w = \left(0, \left[a_w\left(\tilde{K}_S \frac{\partial S}{\partial x'} + \tilde{K}_T \frac{\partial \Theta}{\partial x'} + \tilde{K}_{gr}\right) + \left(a_w \tilde{K}_P + \frac{a_g \nu_g \tilde{K}_v \tilde{M}_w \tilde{P}}{(\Theta+\Theta_1)}\right)\frac{\partial \tilde{P}}{\partial x'}\right]\frac{\partial \Theta}{\partial x'}, 0\right)^{\mathrm{T}};$$

$$\mathbf{B}_w(\mathbf{u}_w) = \begin{pmatrix} \tilde{K}_S & \tilde{K}_{T1} & \tilde{K}_{P1} \\ -\lambda_v \tilde{K}_S & (\tilde{K}_e - \lambda_v \tilde{K}_T) & -\lambda_v \tilde{K}_P \\ 0 & (\tilde{K}_{T1} - \tilde{K}_T) & (\tilde{K}_{P1} - \tilde{K}_{P2}) \end{pmatrix};$$

$$\mathbf{b}_w = \begin{pmatrix} -\tilde{K}_{gr} \\ \lambda_v \tilde{K}_{gr} \\ 0 \end{pmatrix};$$

$$\mathbf{C}_w(\mathbf{u}_w) = \begin{pmatrix} \tilde{K}_S & \tilde{K}_{T1} & \tilde{K}_{P1} \\ -\lambda_v \tilde{K}_S & (\tilde{K}_e - \lambda_v \tilde{K}_T) & -\lambda_v \tilde{K}_P \\ 0 & 0 & 0 \end{pmatrix};$$

$$\mathbf{c}_w = \begin{pmatrix} K_{mw}\left(\dfrac{P_{s0}}{\Theta_0} - \dfrac{F(\Theta)}{(\Theta+\Theta_1)}\right) - \tilde{K}_{gr} \\ a_s(1-\Theta) + \lambda_v \tilde{K}_{gr} \\ \tilde{P} - 1 \end{pmatrix}.$$

Dry region.

$$\mathbf{A}_d(\mathbf{u}_d) = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$a_{11} = \frac{1}{(\Theta+\Theta_1)}\,; \quad a_{12} = -\frac{\tilde{P}_{gv}}{(\Theta+\Theta_1)^2}\,; \quad a_{13} = 0\,;$$

$$a_{21} = 0\,; \quad a_{22} = 1 + \frac{a_g \nu_g \tilde{M}_d \tilde{P}}{(\Theta+\Theta_1)}\,; \quad a_{23} = 0\,;$$

$$a_{31} = \frac{\gamma_{gs}}{(\Theta+\Theta_1)}\,; \quad a_{32} = \frac{\tilde{M}_d \tilde{P}}{(\Theta+\Theta_1)^2}\,; \quad a_{33} = \frac{1}{(\Theta+\Theta_1)}\,;$$

$$\mathbf{D}_d(\mathbf{u}_d) = \begin{pmatrix} \tilde{K}_{gv1} & 0 & \tilde{K}_{gv2} \\ 0 & 1 & 0 \\ 0 & 0 & \tilde{K}_{gv3} \end{pmatrix};$$

$$\mathbf{a}_d = \left(0, \frac{a_g \nu_g \tilde{K}_v \tilde{M}_d \tilde{P}}{(\Theta+\Theta_1)}\frac{\partial \tilde{P}}{\partial x'}\frac{\partial \Theta}{\partial x'}, 0\right)^{\mathrm{T}};$$

$$\mathbf{C}_\mathrm{d}(\mathbf{u}_\mathrm{d}) = \begin{pmatrix} \tilde{K}_{\mathrm{gv1}} & 0 & \tilde{K}_{\mathrm{gv2}} \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \mathbf{c}_\mathrm{d} = \begin{pmatrix} \dfrac{K_{\mathrm{mw}}}{\nu_{\mathrm{gs}}}\left(\dfrac{P_{\mathrm{s1}}}{\Theta_0} - \dfrac{\tilde{P}_{\mathrm{gv}}}{(\Theta+\Theta_1)}\right) \\ a_\mathrm{s}(1-\Theta) \\ \tilde{P}-1 \end{pmatrix}, \tilde{P}_{\mathrm{gv}} > P_{\mathrm{s1}}.$$

SECHAGE PAR CONVECTION D'UNE PLAQUE CONSOLIDEE DE MATERIAU POREUX HUMIDE

Résumé—On formule un modèle pour décrire le mécanisme de séchage d'une plaque agglomérée de matériau poreux humide, par convection forcée d'air chaud sec sur la surface exposée de la plaque. Le modèle décrit l'évolution des distributions de température, pression et humidité à la fois dans les régions humides et sèches. L'avantage du modèle est que les coefficients sont construits à partir des résultats expérimentaux bien connus. On applique le modèle au séchage de briques humides. On résout numériquement par la technique des différences finies le système de trois équations non linéaires aux dérivées partielle qui décrit le mécanisme physique.

KONVEKTIVE TROCKNUNG EINER PLATTE AUS VERDICHTETEM NASSEM PORÖSEM MATERIAL

Zusammenfassung—Es wird ein Modell zur Beschreibung des Trocknungsvorgangs in einer endlich großen gepreßten Scheibe aus nassem porösem Material erstellt. Die Trocknung geschieht durch erzwungene Konvektion mittels heißer, trockener Luft an der Oberfläche. Das Modell ist sehr umfassend und beschreibt das Fortschreiten von Temperatur, Druck und Feuchtigkeitsverteilung sowohl im nassen als auch im trockenen Gebiet. Ein deutlicher Vorteil des Modells besteht darin, daß die Koeffizienten aus gut bekannten experimentellen Ergebnissen berechnet werden. Das Modell wird anhand der Trocknung von nassen Ziegeln erläutert. Das System von drei nichtlinearen gekoppelten partiellen Differentialgleichungen beschreibt den physikalischen Prozeß und wird mittels eines Finite-Differenzen-Verfahrens numerisch gelöst.

КОНВЕКТИВНАЯ СУШКА ЗАТВЕРДЕВАЮЩЕЙ ПЛИТЫ ИЗ ВЛАЖНОГО ПОРИСТОГО МАТЕРИАЛА

Аннотация—Сформулирована модель процесса сушки затвердевающей плиты конечных размеров, изготовленной из влажного пористого материала, при вынужденной конвекции в случае обтекания поверхности пластины потоком сухого воздуха. Модель является достаточно полной и описывает изменение температуры, давления и распределения влаги во влажных и сухих областях. Явным преимуществом модели является то, что коэффициенты строятся по хорошо известным экспериментальным результатам. Применение модели иллюстрируется на примере сушки влажного кирпича. Методом конечных разностей численно решена система трех описывающих физический процесс нелинейных связанных дифференциальных уравнений в частных производных.